5

Exponents (Powers)



You have already studied about integers, rational numbers, and their properties of various operations like addition, subtraction, multiplication, and division. You know that 7 + 7 = 14. The expression 7 + 7 can also be written as 2×7 , read as two times seven. Similarly seven times seven can be written as $7 \times 7 = 49$.

There is another way to express $7 \times 7 = 49$ —that is, $7^2 = 49$. Here 7^2 is read as 7 raised to the power of two, or 7 squared. Similarly $7 \times 7 \times 7 = 343$ can be written as $7^3 = 343$ in which 7^3 is read as 7 raised to the power of three, or 7 cubed.

What is 7^2 or 7^3 ? This is nothing but the exponential form of a number. Exponents are shortcuts for multiplication. The word exponent indicates how many times a number is being multiplied by itself. In 7^2 , 7 is called the base and 2 is the exponent or power.

In 7^3 , 7 is the base and 3 is the power, or exponent.

To illustrate this more clearly, let us look at the following table:

	Repeated multiplication of a number			Power of the product
	$2 \times 2 \times 2 \times 2$	24	2	4
	5×5×5	53	5	3
-	6×6×6×6×6	65	6	5
	7×7	72	7	2
	$a \times a \times \dots m$ times	a^m	а	m

Here a is called the base and m is called the exponent or power, or index.

In the exponential form, the number which is repeatedly multiplied is called the base and the number of times it is repeated is called the exponent, or power or index. This notation of writing the product of a rational number by itself several times is called the exponential notation.

If the base is a negative integer, then the product will be either negative or positive depending upon whether the exponent is an odd number or an even number.

Examples:

$$(-5)^3 = -125$$

(Power is odd, so the product is negative.)

$$(-4)^4 = 256$$

(Power is even, so the product is positive.)

$$(-3)^5 = -243$$

(Power is odd, so the product is negative.)

Example 1: Write the base and exponent of each of the following:

(a)
$$4^3$$

E

Solution:

(a) Base
$$= 4$$

$$Exponent = 6$$

Example 2: Express in power notations:

(a)
$$\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$$
 (b) $\frac{-343}{729}$

(b)
$$\frac{-343}{729}$$

(c)
$$\frac{8000}{27}$$

Solution:

(a)
$$\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{4}{5}\right)^4$$

(b)
$$\frac{-343}{729} = \frac{-7 \times -7 \times -7}{9 \times 9 \times 9} = \left(\frac{-7}{9}\right)^3$$

(c)
$$\frac{8000}{27} = \frac{20 \times 20 \times 20}{3 \times 3 \times 3} = \left(\frac{20}{3}\right)^3$$

Example 3: Find the values of the following:

(a)
$$\left(\frac{2}{5}\right)^{\frac{1}{2}}$$

(a)
$$\left(\frac{2}{5}\right)^4$$
 (b) $\left(\frac{-3}{4}\right)^3$

Solution:

(a)
$$\left(\frac{2}{5}\right)^4 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{16}{625}$$

(b)
$$\left(\frac{-3}{4}\right)^3 = \frac{-3}{4} \times \frac{-3}{4} \times \frac{-3}{4} = \frac{-27}{64}$$

LAWS OF EXPONENTS

The laws of exponents are very useful in performing the operations of multiplication and division.

Law 1: If x is a non-zero rational number and a and b are positive integers, then

$$x^a \times x^b = x^{a+b}$$

If bases are the same, then the powers are added in the multiplication of numbers.

Examples:

(a)
$$5^6 \times 5^4 = (5)^{6+4} = 5^{10}$$

(b)
$$\left(\frac{3}{7}\right)^3 \times \left(\frac{3}{7}\right)^3 = \left(\frac{3}{7}\right)^{3+3} = \left(\frac{3}{7}\right)^6$$

(c)
$$\left(\frac{-2}{5}\right)^2 \times \left(\frac{-2}{5}\right)^5 = \left(\frac{-2}{5}\right)^{2+5} = \left(\frac{-2}{5}\right)^7$$

Law 2: If x is a non-zero rational number and a and b

$$x^a \div x^b = x^{a-b}$$
, when $a > b$

$$x^a \div x^b = \frac{1}{x^{b-a}}$$
, when $b \ge a$

If bases are the same, then the powers are subtracted in the division of numbers.

Examples:

(a)
$$3^6 \div 3^4$$

Here $6 > 4$.
So $3^6 \div 3^4 = 3^{6-4} = 3^2$.

(b)
$$4^3 \div 4^7$$

Here 3 < 7.

So
$$4^3 \div 4^7 = \frac{1}{4^{(7-3)}} = \frac{1}{4^4} = 4^{-4}$$
.

(c)
$$\left(\frac{5}{9}\right)^9 \div \left(\frac{5}{9}\right)^4$$

Here 9 > 4.

So
$$\left(\frac{5}{9}\right)^9 \div \left(\frac{5}{9}\right)^4 = \left(\frac{5}{9}\right)^{9-4} = \left(\frac{5}{9}\right)^5$$
.

Law 3: If x is a non-zero rational number and a is a negative integer, then $x^{-a} = \frac{1}{x^a}$.

Examples:

(a)
$$3^{-7} = \frac{1}{3^7}$$

(b)
$$(-4)^{-2} = \frac{1}{(-4)^2}$$

(c)
$$\left(\frac{5}{9}\right)^{-9} = \frac{1}{\left(\frac{5}{9}\right)^9} = \left(\frac{9}{5}\right)^9$$

Law 4: If x is a non-zero rational number and a and b are positive integers, then $(x^a)^b = x^{ab}$.

Examples:

(a)
$$(3^2)^4 = 3^{2 \times 4} = 3^8$$

(b)
$$\left[\left(\frac{-2}{3} \right)^4 \right]^5 = \left(\frac{-2}{3} \right)^{4 \times 5} = \left(\frac{-2}{3} \right)^{20}$$

Law 5: If x is a non-zero rational number, then $x^0 = 1$.

Examples:

(a)
$$6^4 \div 6^4 = \frac{6 \times 6 \times 6 \times 6}{6 \times 6 \times 6 \times 6} = 1$$

or $6^{4-4} = 6^0 = 1$

(b)
$$13^3 \div 13^3 = \frac{13 \times 13 \times 13}{13 \times 13 \times 13} = 1$$

or $13^{3-3} = 13^0 = 1$

Law 6: If x is a non-zero rational number, then $x^1 = x$

Example:

$$6^{3} \div 6^{2} = \frac{6 \times 6 \times 6}{6 \times 6}$$
or $6^{3-2} = 6$

Law 7: If x and y are non-zero rational numbers and a is a positive integer, then

$$x^{a} \times y^{a} = (xy)^{a}$$

and $x^{a} \div y^{a} = \left(\frac{x}{y}\right)^{a}$

Examples:

(a)
$$3^3 \times 4^3$$

 $3^3 \times 4^3 = 3 \times 3 \times 3 \times 4 \times 4 \times 4$
 $= (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
 $= 12 \times 12 \times 12$
 $= (12)^3 = (3 \times 4)^3$

(b)
$$2^{-2} \times 3^{-2}$$

 $2^{-2} \times 3^{-2} = \frac{1}{2^2} \times \frac{1}{3^2}$
 $= \frac{1}{2 \times 2 \times 3 \times 3}$
 $= \frac{1}{(2 \times 3) \times (2 \times 3)}$
 $= \frac{1}{(2 \times 3)^2}$
 $= \frac{1}{6^2} = 6^{-2} = (2 \times 3)^{-2}$

(c)
$$5^{4} \div 4^{4} = \frac{5^{4}}{4^{4}}$$

$$= \frac{5 \times 5 \times 5 \times 5}{4 \times 4 \times 4 \times 4}$$

$$= \left(\frac{5}{4}\right) \times \left(\frac{5}{4}\right) \times \left(\frac{5}{4}\right) \times \left(\frac{5}{4}\right)$$

$$= \left(\frac{5}{4}\right)^{4}$$

Example 4: If
$$\frac{x}{y} = \left(\frac{3}{7}\right)^3 \div \left(\frac{3}{7}\right)^{-2}$$
, find $\left(\frac{x}{y}\right)^2$.

Solution:
$$\frac{x}{y} = \frac{\left(\frac{3}{7}\right)^{2}}{\left(\frac{3}{7}\right)^{-2}}$$
$$= \left(\frac{3}{7}\right)^{3} \times \left(\frac{3}{7}\right)^{2}$$
$$= \left(\frac{3}{7}\right)^{3+2} = \left(\frac{3}{7}\right)^{5}$$

So
$$\left(\frac{x}{y}\right)^2 = \left[\left(\frac{3}{7}\right)^5\right]^2 = \left(\frac{3}{7}\right)^{5\times 2}$$
$$= \left(\frac{3}{7}\right)^{10}.$$

Example 5: Compare 5² and 2⁵.

Solution:
$$5^2 = 5 \times 5 = 25$$

 $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Hence $2^5 > 5^2$.

Example 6: Express 625 × 10,000 in power notation.

Example 7: Find the value of a if $3^a \times 3^{2a+4} = 3^{13}$.

Solution:
$$3^a \times 3^{2a+4} = 3^{13}$$

$$3^{a+2a+4} = 3^{13}$$

(Using
$$x^a \times x^b = x^{a+b}$$
)

Bases are the same, so powers will be the same on both sides.

$$a + 2a + 4 = 13$$

$$3a + 4 = 13$$

$$3a = 9$$

$$a = 3$$



1. Write the bases and powers of each of the following:

- (b) $\left(\frac{8}{9}\right)^2$
- (c) $(17)^4$
- (d) $\left(\frac{-2}{3}\right)^7$ (e) $\left(\frac{7}{17}\right)^7$

- Expand the following:
 - (a) $a^3b^2c^5$
- (b) $x^2 y^3 z$
- (c) p^2q^3
- (d) $\left(\frac{3}{4}\right)^2$
- (e) $(-9)^3$

3. Express each of the following in exponential forms:

- (a) $(-2) \times (-2) \times (-2) \times (-2)$
- (b) $6 \times 6 \times 6 \times 6 \times a \times a$
- (c) $(-4) \times (-4) \times (-4) \times y \times y$
- (d) $p \times p \times p \times q \times q \times r$

(e) $a \times a \times (-b) \times (-b) \times c$

(f) $\frac{-4}{2} \times \frac{-4}{2} \times \frac{-4}{2}$

4. Write the following in exponential forms:

- (a) $\frac{-27}{343}$ (b) $\frac{1}{125}$
- (c) $\frac{81}{10000}$
- (d) $\frac{64}{729}$
- (e) $\frac{-1}{1000}$

Find the values of the following:

- (a) 6^3
- (b) $(12)^2$
- (c) $\left(\frac{-11}{12}\right)^2$ (d) $\left(\frac{-3}{5}\right)^3$
- (e) $\left(\frac{5}{-7}\right)^2$

6. Compare the following:

- (a) $(-2)^4$ or $(-3)^3$ (b) 7^3 or 6^4
- (c) 4² or 2⁴
- (d) 10^2 or 5^4
- (e) 52 or 25

7. Simplify:

- (a) $(-7)^2 \times 3^2$ (b) $(-12)^2 \times (-4)^3$ (c) $(-2)^3 \times 3^2 \times 5$
- (d) $7^2 \times 2^2 \times (-5)^2$

- (e) $4^2 \times (-5)^3 \times 3$ (f) $4 \times 5 \times 20^2$

- (a) $(-16)^5 \times (-16)^3$ (b) $(18)^4 \times (18)^7$ (c) $\left(\frac{5}{7}\right)^7 \div \left(\frac{5}{7}\right)^7$ (d) $(-41)^6 \div (-41)^3$

- (e) $\left(\frac{3}{5}\right)^3 \div \left(\frac{3}{5}\right)^5$ (f) $\left(\frac{-2}{3}\right)^3 \times \left(\frac{-2}{3}\right)^5$

- Express each of the following with positive integers as exponents:
- (b) $4^7 \div 4^{11}$
- (c) a4 x a-6
- $(d) (4^{-6})^2$

- 10. Express each of the following with negative integers as exponents:
 - (a) $y^2 \times \frac{y^2}{y^4}$ (b) $\frac{3}{y}$
- (c) $\frac{2}{3}$
- (e) $\frac{1}{2^{+}}$

- 11. Find the values of-
 - (a) $5^0 + 7^0$
- (b) $\left[\left(\frac{-3}{5}\right)^3\right]^2$ (c) $(6^0 + 17) \div 2^5$ (d) $(4^{-1} + 5^{-1}) \div 3^2$

- (e) $[(-1)^3]^6$
- (f) $\left[\left(\frac{3}{4} \right)^2 \right]^4 \div \left[\left(\frac{3}{4} \right)^4 \right]^2$
- 12. Find the values of x if—
 - (a) $\left(\frac{-2}{5}\right)^{\circ} \times \left(\frac{-2}{5}\right)^{\circ} = \left(\frac{-2}{5}\right)^{1-1}$
- (b) $\left(\frac{5}{9}\right)^{12} \div \left(\frac{5}{9}\right)^{x} = \frac{125}{729}$
- (c) $\left(\frac{4}{11}\right)^x \times \left(\frac{4}{11}\right)^8 = \left(\frac{4}{11}\right)^{15}$
- (d) $\left(\frac{-3}{4}\right)^x = \left(\frac{-3}{4}\right)^8 \div \left(\frac{-3}{4}\right)^2$

Points to Remember



- If a is a rational number and m is a positive integer, then $a \times a \times a \times ... m$ times = a^m (read as a raised to the power of m), where a is called the base and m is called the exponent, or power or index.
- The laws of exponents state that if x is a rational number and a and b are positive integers, then

$$x^a \times x^b = x^{a+b}$$

$$x^{a} \div x^{b} = x^{a-b}$$
, if $a > b$
= $\frac{1}{x^{b-a}}$, if $b > a$

$$(x^a)^b = x^{ab}$$

$$x^a \times y^a = (x \times y)^a = (xy)^a$$

$$x^{-a} = \frac{1}{x^a}$$

 $x^0 = 1$, where x is not equal to zero.

class-7 Syb. - maths chapter-5 Exponents (Powers) Page NO. 1-48,0 / If a is a grational number and m is a positive integer, then axaxaxax --- xm times = am Here a is called the base and m is called the exponent or poner or index In the enponential form, the number which is repeatedly multiplied is called the base and the number of times it is repeated is called the exponent, or power or inder This notation of nexiting the product of a rational number by itself several times is called the exponential notation. & Laws of Enponents: hand I: - If it is a non-zero rational number and a and b are positive integers, then xaxxb = xa+b It bases are the same then the Poners are added in the multiplication of numbers. x q - y q

Sale mather she Land 2: If x is a non-zero rational number and a and b are positive integers, then xa = xb = xa-b when a>b 29 = 26 = 1 when 6>9 If bases are the same, then the Powers are subtracted in the division of numbers Land 3:- If n is a non-zero rational number and a is a negative integers then x 9= 1 Lans 4! - (x9) = xab Law 5: - x = 1 Example :- 60 = 1 Long 6: = x = x Low 7: - If x and y are non-zero rational numbers and a is a

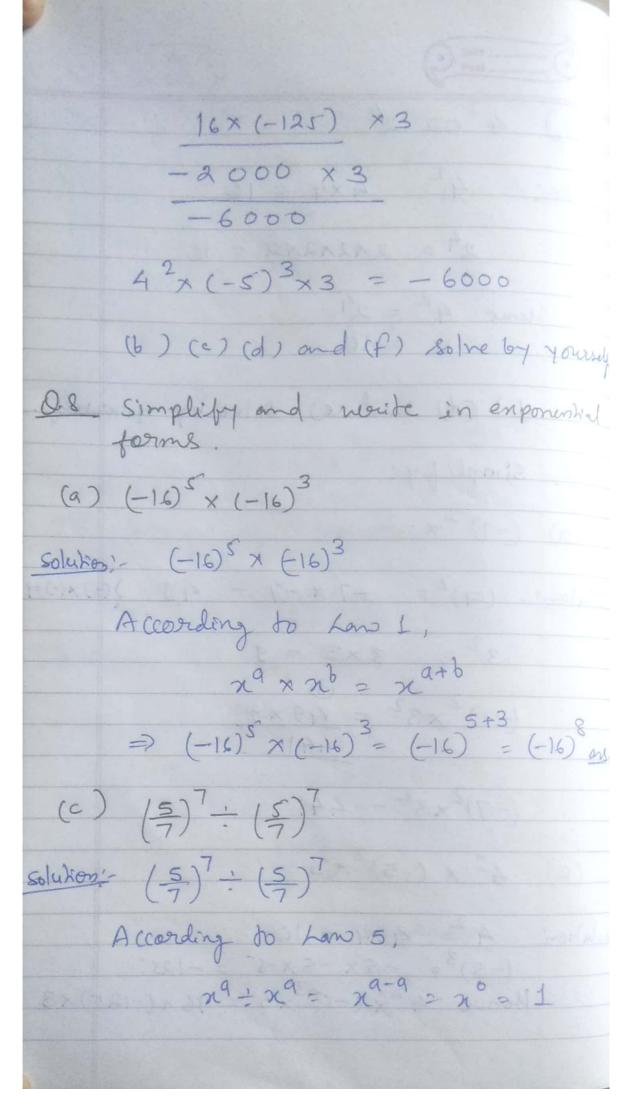
Page NO. - 51 Ex. 5.1 class - 7 Q.L write the bases and Powers of each of the following: (a) (-4) 6 (b) (8) 2 (c) (17) 4 (d) (-2) 4 (e) (7) solution: (a) (-4)6 Here, Base = (-4) and (b) (8)² Here, Base = (8) and Poner = 2 (c) (d) and (e) solve by yourself as somed above. Q.2 Expand the following: (9) 9362(5 (b) x2y3z (c) p2g3 (d) $(\frac{3}{4})^2$ (e) $(-9)^3$ solution'- (a) a3b2c5 = axaxaxbxbxcxcxcxcxc $(d) (\frac{3}{4})^2 = \frac{3}{4} \times \frac{3}{4}$ (e) $(-9)^3 = (-9) \times (-9) \times (-9)$

	Lans Chille
) (c) solve by yourself.
0.3 E.	apress each of the following enponential forms:
(a) (-	$-2) \times (-2) \times (-2) \times (-2)$
Solution:	$(-2)^4$
(c) E	-4) x (-4) x (-4) x y x y
Solution! -	$(-4)^3 \times y^2$
(f) ($-\frac{4}{3}$ \times $\left(-\frac{4}{3}\right)$ \times $\left(-\frac{4}{3}\right)$
solution!	(+4) ³ (8)
) (d) and (e) solve by yourself.
0.4 W	brite the following in enponential
(a) -	343
Solukasi	-27 $3[27$ $7[343]$ 319 $7[49]$
DX	3 3 7 49
	$\frac{27}{343} = \frac{-3 \times -3 \times -3}{7 \times 7 \times 7} = \left(\frac{-3}{7}\right)^3$
the same of the sa	343 7×7×7 (7)

(c) <u>81</u>	
solution; - 81 3/8	1 10/10000
10000	7 10/1000
3/3	
	10(10
$\frac{81}{10000} = \frac{3 \times 3 \times 3}{10 \times 10 \times 10}$	-/
(d) 64 729	
solution: - 64 4/64	9 1729
729 4/16	9[8]
7.4	
64 = 4×4×6	$\frac{1}{9} = \left(\frac{4}{9}\right)^3$
$729 = 9 \times 9 \times$ (b) and (e) 80	100 100 - 2011
1673 + G3xG-3 (
QE find the value	of the following
(a) 6 ³ (8) × (8) ×	
CD CAXCAY	
solution: 63 = 6×6×6	
$= 36 \times 6$ $= 216$	
$6^3 = 216$ ar	& A Something
(c) 1-11)2	
$(c) \left(-\frac{11}{12}\right)^2$	

$\frac{\text{solution}}{(-1)^2} = \frac{-11 \times -11}{12 \times 12}$
$= \frac{121}{144}$
$(e) \left(\frac{5}{-7}\right)^2$
$\frac{\text{Solution}}{(-7)^2} = \frac{5 \times 5}{-7 \times -7}$
Q.6 Compare the following:
$(9) (-2)^4 091 (-3)^3$
Solution! - (-2)4= (-2)x(-2)x(-2)x(-2)
$= 16$ $(-) \times (-) = (+)$
$(-3)^{3} = (-3) \times (-3) \times (-3)$
$= -27$ Hence $(-2)^4 > (-3)^3$
16 > (-27)

(c) 42 or 24 solution - 42 = 4x4 = 16 24 = 2×2×2×2 = 16 Henre 42 = 24 16 = 16 (b) (d) and (e) solve by yourself. Q.7 Simplify: (a) $(-7)^2 \times 3^2$ $solution! = (-7)^2 = -7 \times -7 = 49 = (-7) \times (-7) = 1$ $3^2 = 3 \times 3 = 9$ $(-7)^2 \times 3^2 = 49 \times 9$ 441 $(-7)^2 \times 3^2 = 441$ (e) $4^2 \times (-5)^3 \times 3$ Solution: $4^2 = 4x4 = 16$ $(-5)^3 = -5x - 5x - 5^2 = -125^2$ Hence 42x (-5)3x3 = 16x(-125)x3



$$(f) \frac{(3)^{7}}{(3)^{3}} \times (\frac{3}{7})^{6} = (\frac{5}{7})^{7-7} = (\frac{5}{7})^{6} = 1 \text{ and.}$$

$$(f) \frac{(-2)^{3}}{(3)^{3}} \times (\frac{-2}{3})^{6}$$
Solution:
$$(\frac{-2}{3})^{3} \times (\frac{-2}{3})^{6}$$
According to have 1,
$$n^{9} \times n^{1} = n^{4+6}$$

$$(\frac{-2}{3})^{3} \times (\frac{-2}{3})^{6} = (\frac{2}{3})^{3+6} = (\frac{2}{3})^{3}$$

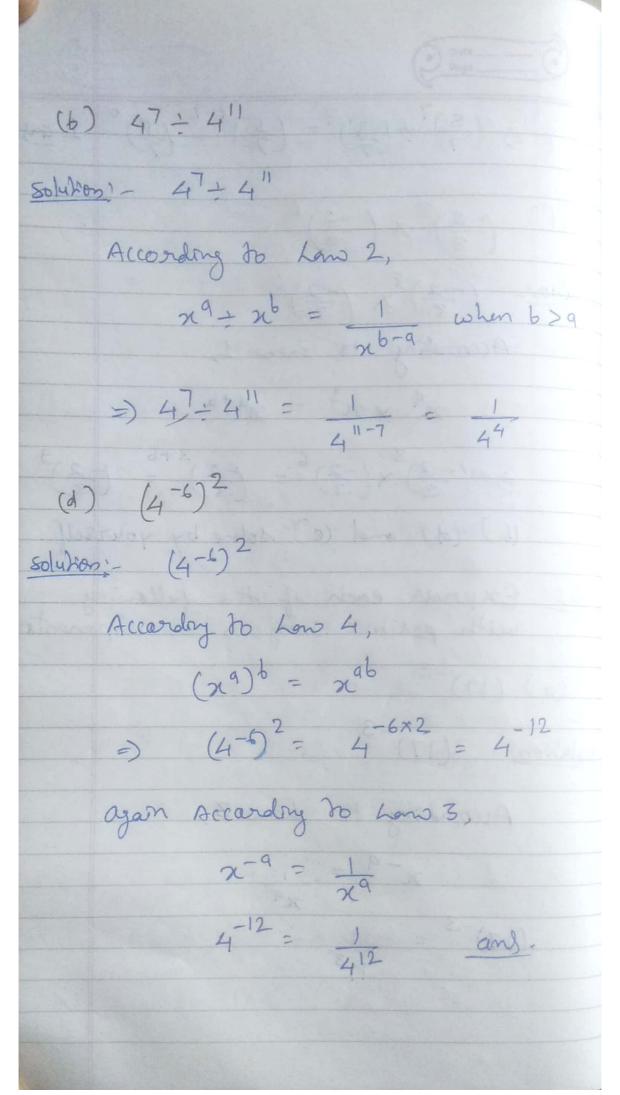
$$(6) (d) \text{ and (e) solve by yourself.}$$

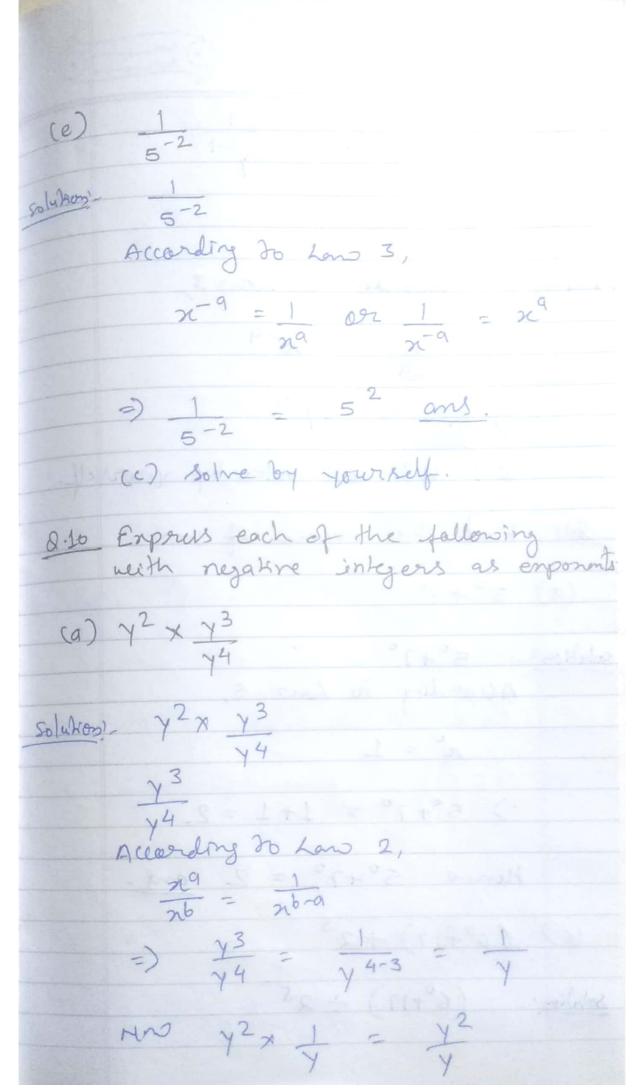
$$(a) (d) \text{ and (e) solve by yourself.}$$

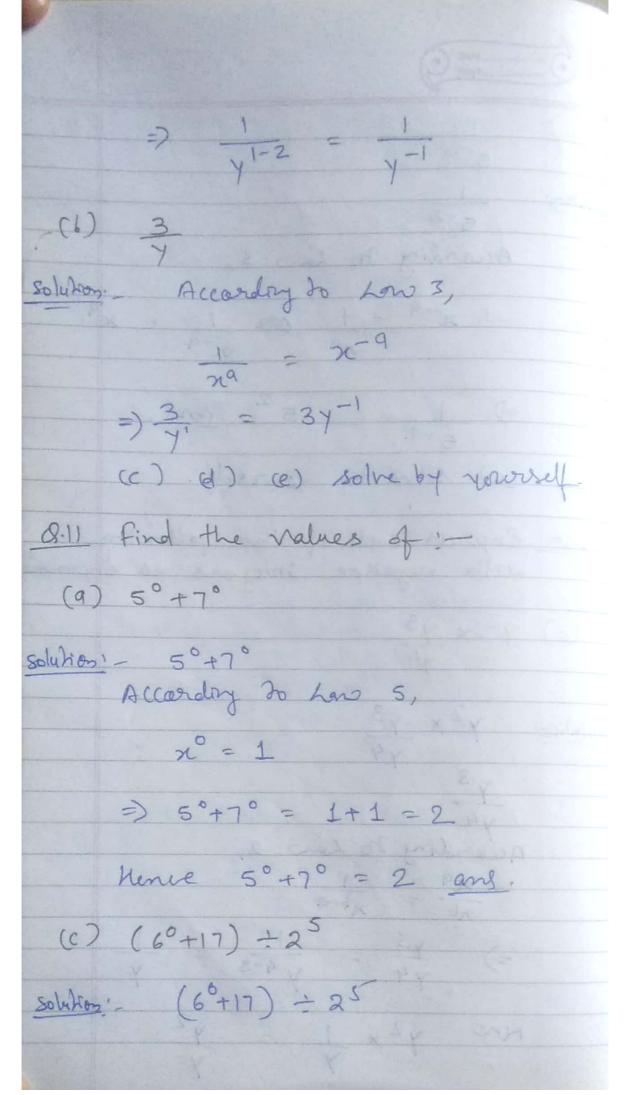
$$(a) (17)^{-3}$$

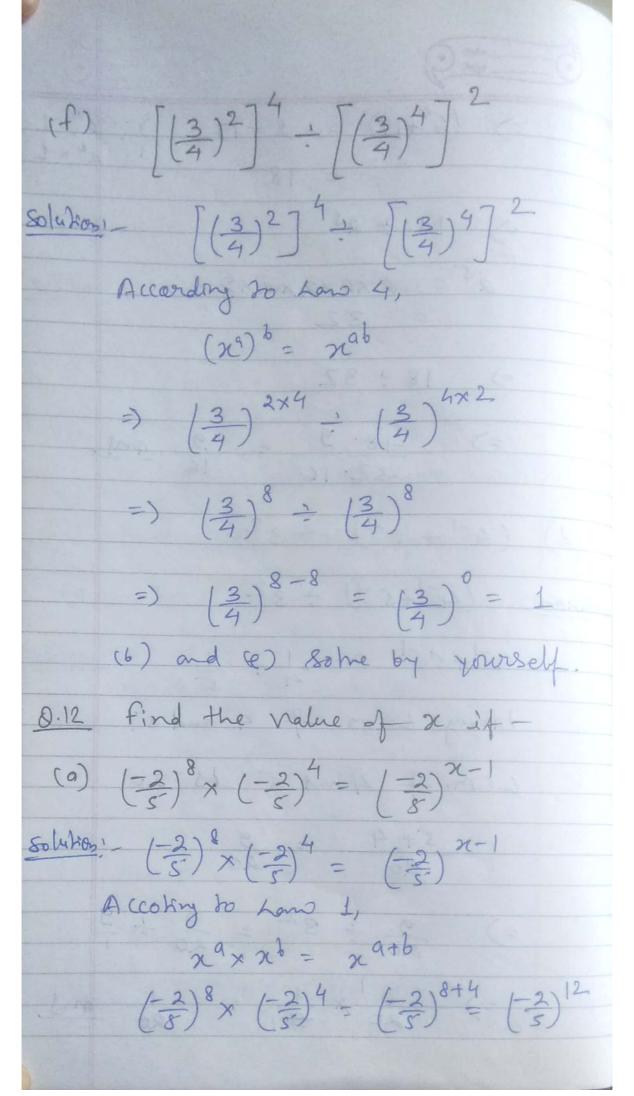
$$(a) (17)^{-3}$$
Solution:
$$(a) (17)^{-3} = \frac{1}{17^{3}}$$

$$(17)^{-3} = \frac{1}{17^{3}}$$









$\frac{1}{(-2)^{12}} = \frac{(-2)^{2}}{5}$
Bases are the same, so powers will be the same on both sides.
$=) 2 = \chi - $
=) $12+1=x$
=> x = 13
Lenzielle de la controlle de l
(b) $\left(\frac{5}{9}\right)^{12} + \left(\frac{5}{9}\right)^{2} = \frac{125^{2}}{729}$
solukan' - 125° = 57125° 91729 729 = 125° 9181
729 5/25 9/81
THE PARTY OF THE P
$\frac{125}{729} = \frac{5 \times 5 \times 5^2}{9 \times 9 \times 9} = (\frac{5}{9})^3$
$=\frac{15}{9}$ $=\frac{5}{9}$ $=\frac{5}{9}$ $=\frac{5}{9}$
According to 6m2 2.
na = no = na-b
$=$ $(\frac{5}{9})^{12}$ $(\frac{5}{9})^{12}$ $(\frac{5}{9})^{12-24}$
$=$ $\left(\frac{5}{9}\right)^{12-21} = \left(\frac{5}{9}\right)^3$
Bases are the same, so powers

will be the same on both woles =) 9 = x and (c) (4) × (4) = (4) 15 solution! (4) x (4) 8 = (4) 15 According to Land 1, naxnb=naxb. =) (4) × (4) 8 = (4) × +8 =) (4) NT8 = (4) 15 - Bases are the same, so power well be some on both edds = S X+8 = 15 => 15-8 (d) solve by yourself.